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Analytical Development of an Equivalent System Mismatch Function

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A mismatch function is used to check the validity of a low-order equivalent system model that has been derived from a high-order system representation. If the difference between the low- and high-order models is greater than allowed by the mismatch function, the flying qualities predictions obtained from parameters of the low-order equivalent system may not be representative of the ratings a pilot would give the actual aircraft. A methodology is developed in this paper to derive equivalent system mismatch functions analytically. The methodology is used to analytically determine a mismatch function for the longitudinal axis of a class IV fighter aircraft in the category A, nonterminal flight phase.

Introduction

TO predict the flying qualities characteristics of a given aircraft, the current military flying qualities specification recommends the use of an equivalent systems model of the airplane.[1] Much of the experience and flying qualities information embodied in these specifications was gathered before complicated electronic stability augmentations systems were commonly used on aircraft. The stability augmentation systems offer the potential to modify the flight characteristics of the airplane significantly. However, the response of the augmented aircraft may no longer be dominated by model responses that are easily identified in the classical form, such as the phugoid and short-period modes. In fact, the order of the augmented aircraft model can be much greater than that of the traditional fourth-order form. The equivalent system method attempts to approximate the high-order model of the augmented aircraft by a low-order (usually fourth-order) form that is consistent with the notation and experience compiled in the flying qualities specifications.[2]

To assess the quality of the low-order equivalent system approximation, the mismatch function shown in Fig. 1 can be used. The mismatch function is an approximate measure of the maximum unnoticeable dynamics. Thus, if the difference between the high-order model of the aircraft and the low-order equivalent system model of the aircraft passes outside of the envelope, it is expected that the pilot would "notice" the difference and perhaps give a different flying qualities rating than that predicted by comparing the parameters of the equivalent system model to the specification.

The envelopes shown in Fig. 1 were obtained empirically by examining pilot rating differences between pairs of configurations flown using the variable stability NT-33 aircraft. Each pair of configurations consisted of an unaugmented, low-order response and a high-order augmented system response obtained by adding terms in series with the low-order response. Augmented dynamics that resulted in one pilot rating change (Cooper-Harper scale) were used to define the envelopes.

Obviously, mismatch function envelopes like that shown in Fig. 1 will be very expensive to develop empirically for different response, aircraft types, and flight conditions. To address this problem, a new methodology is put forth in this paper that develops mismatch functions analytically. The method focuses on the effect of mismatch on the performance of the closed-loop pilot/vehicle system. A new mismatch function will be derived in this paper and will be compared to Fig. 1.

Interpreting Mismatch as Dynamic Uncertainty

Bacon and Schmidt described the fundamental nature of equivalent system modeling using Fig. 2 (Ref. 3). Figure 2 depicts a compensatory tracking task wherein the pilot, represented by the transfer function matrix $P(s)$, attempts to control a low-order equivalent system representation of the aircraft, given by $G_{\mathrm{LOS}}(s)$. If the low-order equivalent system is truly representative of the high-order system dynamics, denoted $G_{\mathrm{HOS}}(s)$, then the pilot should not be able to tell whether the switch in Fig. 2 is open or closed. In other words, a given set of pilot dynamics $P(s)$ should result in the same closed-loop performance of the pilot/vehicle feedback system whether $G_{\mathrm{LOS}}(s)$ or $G_{\mathrm{HOS}}(s)$ is being controlled.

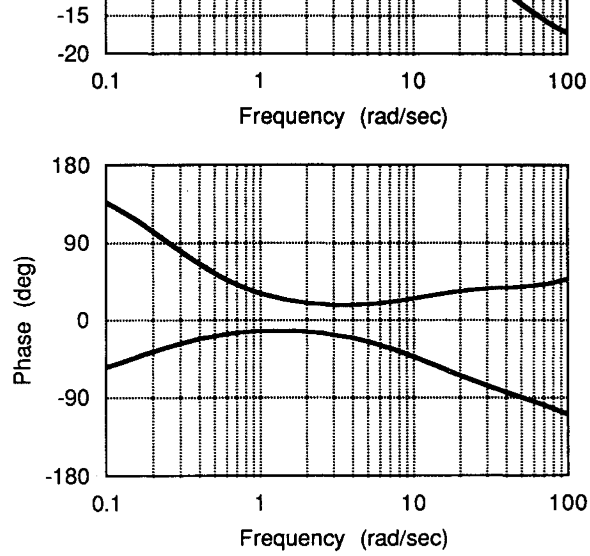

Fig. 1 Military specification mismatch envelope.

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Now consider the feedback diagram shown in Fig. 3. Again, the pilot is represented by $P(s)$ and the low-order equivalent system is represented by $G_{LOS}(s)$. The block $\Delta(s)$ is given by

$$\Delta(s) = \rho\left[G_{HOS}(s) - G_{LOS}(s)\right]G_{LOS}^{-1}(s) \quad (1)$$

where $G_{LOS}(s)$ is assumed to be invertible for convenience. As the parameter ρ varies from zero to one, the controlled element varies from the low-order equivalent system model ($\rho = 0$) to the high-order model ($\rho = 1$). The parameter ρ acts like a continuous equivalent of the discrete on-off switch shown in Fig. 2. Thus, the fundamental objective of equivalent system analysis is to find an equivalent system model such that the performance of the feedback system shown in Fig. 3 does not change (or changes only slightly) as ρ varies from zero to unity.

Pilot modeling studies have shown that the human pilot will alter his equalization to yield good stability and performance characteristics in the pilot/vehicle feedback system. This assumption motivated the development of the well-known crossover model of the human operator.[4] Closed-loop performance of the pilot/vehicle system depicted in Fig. 3 will be used to judge the suitability of the low-order equivalent system model. In other words, a low-order equivalent system model will be considered valid if the closed-loop performance provided by pilot control will not degrade as ρ is varied from zero to unity.

In recent control theory literature, the block in Fig. 3 described by $\Delta(s)$ is called an output multiplicative uncertainty.[5] The $\Delta(s)$ block refers to variations in the nominal controlled element dynamics, which is typically called model uncertainty. Thus, the difference between the high-order system model and its low-order equivalent system model can be considered a source of uncertainty in the feedback diagram.

Performance robustness is the term frequently used to describe the effects of uncertainty on feedback system performance. The pilot's performance objective in the feedback system depicted in Fig. 3 is to make the response from commands (y_c) to error (e) small. Therefore, performance of the pilot/vehicle closed-loop system can be assessed using the transfer function matrix from y_c to e.

$$e(s) = \left\{I + \left[I + \Delta(s)\right]G_{LOS}(s)P(s)\right\}^{-1} y_c(s) \quad (2)$$

When ρ is equal to zero, the transfer function matrix from the command vector to the error vector is

$$e(s) = S(s)y_c(s) \quad (3)$$

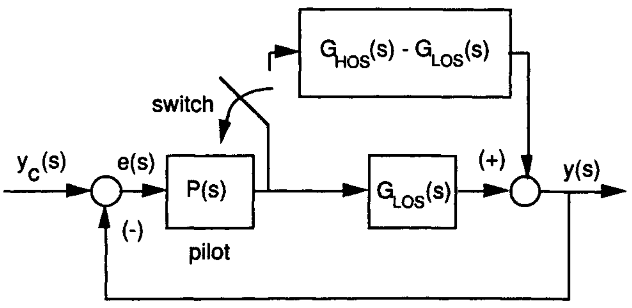

Fig. 2 Closed-loop pilot/vehicle system.

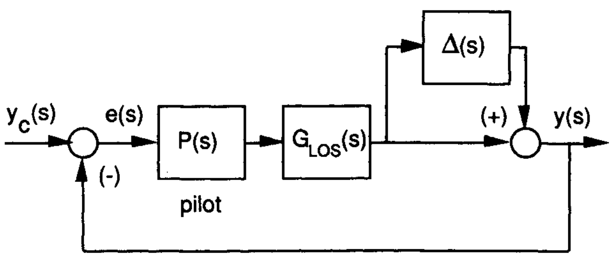

Fig. 3 Uncertainty block diagram.

where $S(s) = [I + G_{LOS}(s)P(s)]^{-1}$ is the sensitivity function of the pilot/vehicle system including the low-order equivalent system model.

The amount of closed-loop performance degradation determines the largest allowable mismatch. Consequently, a vector d will be defined as the deviation in the tracking error signal from the case when ρ is equal to zero. In other words, the degradation vector d is defined as the difference between Eqs. (2) and (3),

$$d(s) = \left(\left\{I + \left[I + \Delta(s)\right]G_{LOS}(s)P(s)\right\}^{-1} - \left[I + G_{LOS}(s)P(s)\right]^{-1}\right) y_c(s) \quad (4)$$

Equation (4) can also be written as

$$d(s) = S(s)\left\{\left[I + \Delta(s)T(s)\right]^{-1} - I\right\} y_c(s) \quad (5)$$

where $T(s) = I - S(s)$ is the complimentary sensitivity function.

Note that when ρ is equal to zero, the performance degradation vector d is also equal to zero. Ideally, no performance degradation should occur as ρ is varied from zero to unity. To quantify the performance degradation, the infinity norm of the transfer function from y_c to d will be used. The transfer function infinity norm, denoted $\|(\cdot)\|_\infty$, is defined for a transfer function matrix $F(s)$ as

$$\|F(s)\|_\infty = \sup_\omega \bar{\sigma}\left[F(j\omega)\right] \quad (6)$$

where $\bar{\sigma}[\cdot]$ denotes the maximum matrix singular value.[6] Therefore, one measure of the performance robustness of the pilot/vehicle feedback system due to mismatch uncertainty is given by

$$\|S(s)\{[I + \Delta(s)T(s)]^{-1} - I\}\|_\infty < \kappa \qquad (0 \le \rho \le 1) \quad (7)$$

where the parameter κ is used to represent the largest admissible (unnoticeable) performance degradation.

It is interesting to note from Eq. (7) that the performance degradation due to equivalent system mismatch will depend on the sensitivity and the complimentary sensitivity functions of the pilot/vehicle system. Therefore, performance degradation depends on the nominal performance of the closed-loop pilot/vehicle system, under the assumption that the pilot is flying a low-order representation of the aircraft. Previous flying qualities research has shown that the achievable closed-loop system performance is strongly influenced by the flying qualities characteristics of the aircraft. Neal and Smith[7] as well as Bacon and Schmidt[8] have explored the relationship between a resonant peak of the complimentary sensitivity function and flying qualities rating levels for longitudinal axis tracking tasks. This relationship is also supported by observations that poor equivalent system matches are generally associated with poor pilot ratings.[2] As a consequence, different values of admissible performance degradation, represented by the value of κ, will be needed for different flying tasks and different nominal performance levels. In other words, one value of κ might be needed to limit performance degradation due to mismatch in an equivalent system model that is predicted to have level I flying qualities while another value is needed for level II equivalent system models.

Development of a New Mismatch Function

To illustrate the analytical development of a mismatch function using Eq. (7), the in-flight simulation conducted by Neal and Smith using the NT-33 aircraft will be considered.[7] Thus, the final results can be compared directly to the empirical result shown in Fig. 1.

Table 1 lists parameters of the high-order pitch rate response transfer functions for five of the Neal-Smith configurations that were all predicted to have level I flying qualities. Also shown in Table 1 are low-order equivalent systems models of

Table 1 Configurations with level I predicted ratings

Config. no.	Pilot rating	$1/\tau_{\theta 2}$, 1/s	ζ_{sp}	ω_{sp}, 1/s	$1/\tau_1$, 1/s	$1/\tau_2$, 1/s	ω_3, 1/s	K_e, 1/lb·s	ζ_e	ω_e, 1/s	τ_d, s
2A	4.5	1.25	0.70	4.9	2.0	5.0	63.0	1.08	0.44	5.97	0.0
2B	4–6	1.25	0.70	4.9	2.0	5.0	16.0	1.07	0.42	5.68	0.06
1B	3.5	1.25	0.69	2.2	2.0	5.0	63.0	1.04	0.67	3.05	0.0
2C	3.0	1.25	0.70	4.9	5.0	12.0	63.0	1.02	0.64	6.07	0.0
2D	2.5	1.25	0.70	4.9	∞	∞	63.0	1.00	0.70	4.89	0.02

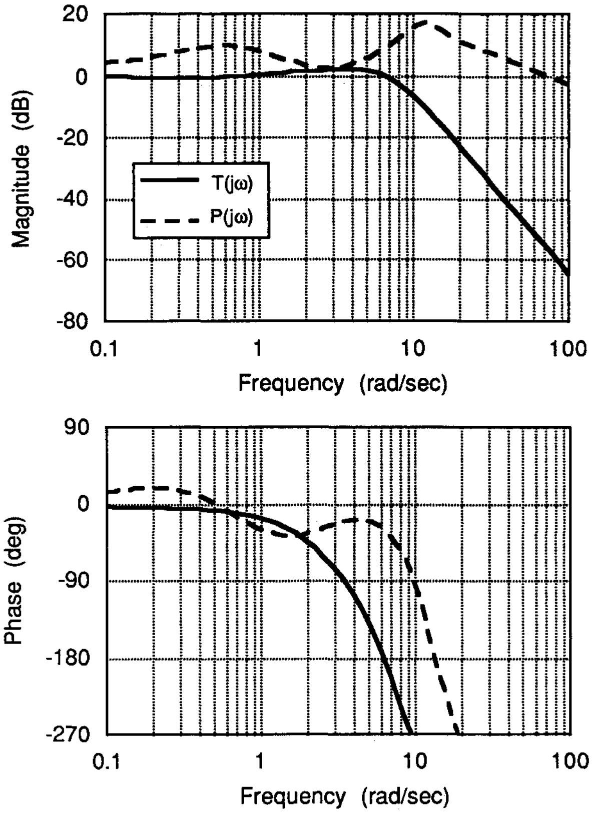

Fig. 4 Configuration 1B frequency response.

each configuration as well as the actual flying qualities ratings obtained during flight testing. The high-order system form for each test configuration is

$$\frac{\dot{\theta}}{F_s} = \frac{(\tau_{\theta 2} s + 1)}{\left[(s/\omega_{sp})^2 + (2\zeta_{sp}/\omega_{sp})s + 1\right]} \times \frac{(\tau_1 s + 1)}{(\tau_2 s + 1)\left[(s/\omega_3)^2 + (1.5/\omega_3)s + 1\right]} \tag{8}$$

where $\dot{\theta}$ is the aircraft pitch attitude rate and F_s is the longitudinal stick force. The assumed low-order equivalent system form is

$$\frac{\dot{\theta}}{F_s} = \frac{K_e(\tau_{\theta 2} s + 1)e^{-\tau_d s}}{\left[(s/\omega_e)^2 + (2\zeta_e/\omega_e)s + 1\right]} \tag{9}$$

The low-order equivalent system models were obtained by minimizing the following cost function

$$J = \frac{1}{20}\sum_{k=1}^{20}\Big\{\left[\text{gain}_{\text{HOS}}(j\omega_k) - \text{gain}_{\text{LOS}}(j\omega_k)\right]^2 + 0.01745\left[\text{phase}_{\text{HOS}}(j\omega_k) - \text{phase}_{\text{LOS}}(j\omega_k)\right]^2\Big\} \tag{10}$$

over 20 frequency points ω_k logarithmically spaced over the interval from 0.1 to 10 rad/s. The "gain" in Eq. (10) is in decibels whereas "phase" is in degrees. Note that the pitch numerator parameter $\tau_{\theta 2}$ is the same for both the high-order and low-order equivalent system models.

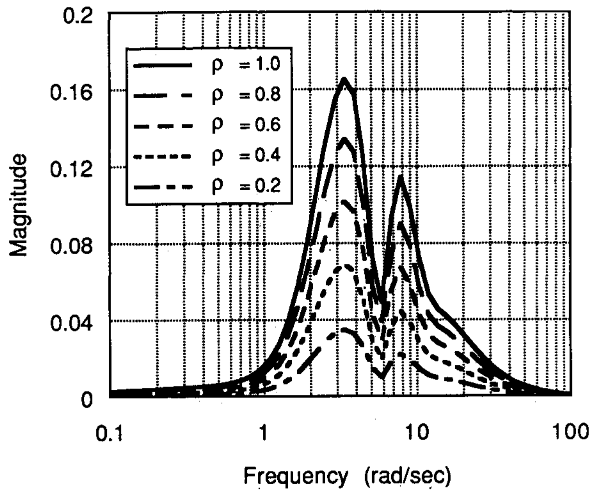

Fig. 5 Configuration 1B performance degradation.

The five configurations were selected because each configuration is predicted to have level I flying qualities. However, the actual pilot ratings for configurations 2A and 2B were in the level II category. This result has been coined the "lead effect" because both of these configurations have augmentation such that additional lead or lag occurs near the short period frequency.[9]

The optimal control pilot model (OCM) was used to generate a frequency-dependent describing function of the pilot $P(j\omega)$, such that the complimentary sensitivity function of the pilot/vehicle system could be computed.[10] The parameters that describe the OCM model were chosen identical to those used by Bacon and Schmidt in their study of the Neal-Smith flight data.[8] Namely, the observation delay was chosen as 0.2 s, the neuromuscular lag time constant as 0.1 s, and the observation and motor noise variances as −20 and −25 dB, respectively. The cost function was a weighted blend of pitch rate and pitch attitude errors while a second-order coloring filter was used to model the commanded response. The frequency response of the pilot describing function and the closed-loop complimentary sensitivity function for configuration 1B are shown in Fig. 4.

Configuration 1B was predicted to have level I flying qualities ratings. The actual rating was 3.5, or exactly on the boundary between levels I and II. None of the equivalent system parameters for 1B were near their respective level I boundaries. This observation leads to the reasonable conclusion that the mismatch uncertainty associated with configuration 1B is about the largest that can be tolerated by the pilot before his perception of true flying qualities of the aircraft is affected by the mismatch.

Figure 5 shows the performance degradation that occurs in configuration 1B when ρ is varied from zero to unity. The curves shown in Fig. 5 represent the maximum matrix singular value of the performance degradation transfer function matrix as a function of frequency. The peak value of the singular value curves yield the value of the transfer function infinity norm.

From Fig. 5, one can see that the infinity norm of the performance degradation transfer function for configuration 1B is about 0.17 (at about 3.5 rad/s) when ρ is equal to unity. Consequently, it will be assumed that the value of the transfer function infinity norm must be less than $\kappa = 0.18$ to insure that the pilot will rate the flying qualities of the aircraft similar to predictions based on the low-order equivalent system.

Figure 6 illustrates the performance degradation of all five Neal-Smith configurations when ρ is equal to unity. Figure 6 reveals that configurations 2A and 2B both have transfer function infinity norm values of 0.24 at about 2.5 rad/s. A value much greater than $\kappa = 0.18$ leads to the conclusion that the parameters of the low-order equivalent system models for configurations 2A and 2B will not accurately predict the flying qualities characteristics of the associated high-order models. This result is consistent with the flying qualities ratings listed in Table 1 where the flying qualities of both configurations 2A and 2B were predicted inaccurately.

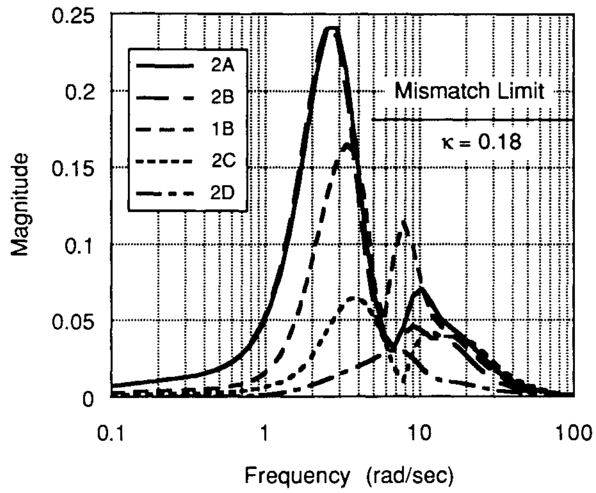

Fig. 6 Performance degradation comparison.

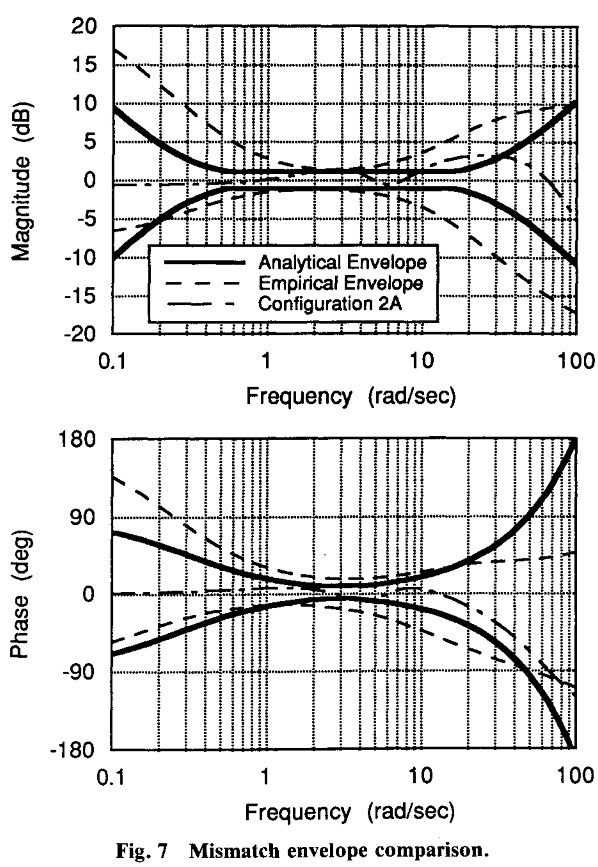

Fig. 7 Mismatch envelope comparison.

A mismatch test like that shown in Fig. 6 is useful in that it accurately predicts when an equivalent system mismatch is unacceptably large. However, a pilot describing function is required to perform the calculations. A more useful description of the allowable mismatch (and one which can be compared to Fig. 1) can be obtained by defining a different uncertainty description than the output multiplicative form. Let $L(s)$ be defined such that

$$\Delta(s) = L(s) - I \tag{11}$$

When $\rho = 0$, Eqs. (1) and (11) reveal that $L(s) = I$. But when $\rho = 1$, $\Delta(s)$ becomes

$$\Delta(s) = \left[G_{\mathrm{HOS}}(s) - G_{\mathrm{LOS}}(s)\right] G_{\mathrm{LOS}}^{-1}(s) \tag{12}$$

and $L(s)$ is given by

$$G_{\mathrm{HOS}}(s) = L(s) G_{\mathrm{LOS}}(s) \tag{13}$$

Equation (13) reveals that the transfer function matrix $L(s)$ represents dynamics that have been "added" to the low-order model $G_{\mathrm{LOS}}(s)$ to yield the high-order model $G_{\mathrm{HOS}}(s)$. The Neal-Smith flight test configurations were formed in much the same manner.

By replacing Eq. (11) in Eq. (7), the closed-loop performance requirement becomes

$$\left\| S(s)\left(\left\{I + \left[L(s) - I\right] T(s)\right\}^{-1} - I\right) \right\|_{\infty} < \kappa \quad (0 \leq \rho \leq 1) \tag{14}$$

To determine admissible representations of $L(s)$, parameters of several simple transfer function models were varied until Eq. (14) was violated. By modeling $L(s)$ by simple transfer function forms, magnitude as well as phase information can be obtained. The transfer function forms are shown in Table 2 along with the resulting parameter values which defined the largest possible mismatch for Neal-Smith configuration 2C with $\rho = 1$ and $\kappa = 0.18$. Configuration 2C was chosen as having closed-loop performance representative of a configuration with level I flying qualities characteristics. The final mismatch envelopes are obtained by computing the frequency responses of each of the transfer function forms in Table 2 and forming an envelope around the resulting plots.

Figure 7 shows the new analytical mismatch boundaries in relation to the empirical boundaries suggested by the current military specifications (Fig. 1). Note that both sets of curves have the characteristic of smaller allowable mismatch near the crossover frequency range of 1.0–10.0 rad/s. The magnitude envelopes have the same general shape but the empirical boundary is much wider, especially at higher frequencies. The phase envelopes are also very similar but the analytical boundaries are larger than the empirical boundaries at very high frequency.

Figure 7 also shows the mismatch error for configuration 2A. Recall that configuration 2A was predicted to have level I flying qualities but was actually rated level II. The mismatch error lies inside the empirical boundaries which implies that the equivalent system model will predict the flying qualities characteristics of the actual aircraft. However, this figure also reveals that the mismatch of configuration 2A passes outside of the new analytical boundary. Thus, the new analytical boundaries would lead to the correct conclusion that the characteristics of the low-order equivalent system model for configuration 2A will not accurately predict the flying qualities of the actual aircraft.

Example Application

To demonstrate one possible use of the analytical mismatch envelopes, consider the equivalent system models reported in Ref. 11 for the F-14 fighter aircraft in Mach 0.5 cruise at 15,000 ft. A high-order, longitudinal axis model of the F-14 at this flight condition, including feel system dynamics, has 11

Table 2 Mismatch boundary transfer functions

Transfer function form	Parameter range
$L(s) = aI$	$0.87 \le a \le 1.13$
$L(s) = [1/(bs+1)]I$	$b \le 0.034$ s
$L(s) = (cs+1)I$	$c \le 0.031$ s
$L(s) = Ie^{ds}$	$-0.033 \le d \le 0.031$ s
$L(s) = [s/(s+f)]I$	$f \le 0.31$ 1/s
$L(s) = [(s+g)/s]I$	$g \le 0.28$ 1/s
$L(s) = Ie^{j\phi}$	$-6.4 \le \phi \le 8.3$ deg

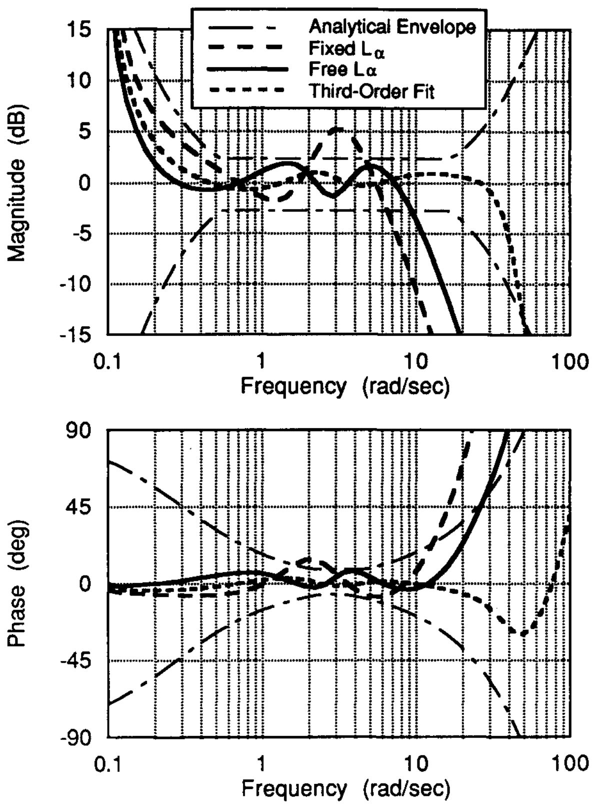

Fig. 8 F-14 mismatch comparison.

states. Second-order equivalent system models like Eq. (9) were obtained in Ref. 11 by minimizing Eq. (10) and either fixing the value of $L_\alpha = 1/\tau_{\theta 2}$ at the high-order model value or allowing it to be freely chosen during minimization. The parameters of the L_α fixed model predicted a flying qualities rating of level II whereas the parameters of the L_α free model predicted a flying qualities rating of level III. A third equivalent system model was also obtained by allowing an additional real pole in the equivalent system model Eq. (9), resulting in a third-order model.

Figure 8 depicts the mismatches for the three equivalent system model representations of the F-14 aircraft. The dashed line reveals that the L_α fixed equivalent system model mismatch passes outside of the analytical mismatch envelope several times in the frequency range from 0.3 to 10.0 rad/s (the frequency range used in developing the equivalent system model). The mismatch represented by the thick solid line shows that the analytical magnitude boundary is violated at about 9 rad/s for the L_α free equivalent system model. In violating the mismatch envelopes, neither the L_α fixed nor the L_α free second-order equivalent system models are expected to accurately predict the flying qualities rating of this aircraft configuration.

By allowing an additional pole in the equivalent system model, an improved match is obtained. The dotted line in Fig. 8 reveals that the analytical mismatch envelope is violated in magnitude only for frequencies greater than about 50.0 rad/s when a third-order equivalent system model is used. Although the mismatch envelope is violated by the third-order equivalent system model, the third-order model is clearly a better representation of the high-order model than either of the second-order models. A third-order equivalent system model does not comply with the current military flying qualities specifications; however, Bischoff was able to make reasonable flying qualities predictions by defining an "effective" short period natural frequency that takes into account the additional real pole.[11]

Conclusion

Analysis of the Neal-Smith NT-33 in-flight simulation test data suggests a link between large equivalent system mismatch and erroneous flying qualities predictions using equivalent system model parameters. This research has shown that an equivalent system mismatch function can be developed analytically. Thus, the fidelity of an equivalent system model can be tested to confirm the accuracy of subsequent flying qualities analysis.

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